The "Rule of 500" and Guiding Tolerance Depend on Size of the Finished Picture, Not Just Lens and Sensor

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Defining the problem

The "Rule of 500" says that you can expose up to $500/F$ seconds when photographing the stars with a fixed tripod; if you go longer than that, the stars will look like streaks due to the earth's rotation. Is that true? Is it still valid in the digital era?

To find out, let's do the math, from the beginning.

First we have to define what it means not to look like a streak. It's reasonable to say that an acceptable amount of stretching of star images is some number of pixels (probably 1) in the *finished* picture, not on the sensor. Let's call this quantity S.

That point is important. It doesn't matter how many *sensor* pixels the star is smeared across, if you can't see the smearing in the finished picture. And we normally view pictures at much less than the full resolution of the sensor.

That means we also need to know the width of the finished picture in pixels; call that W_P. A small picture on a web page is typically 600 pixels wide. A 4×6-inch print is 900 pixels wide if it achieves 150 dots per inch, which is

S	acceptable amount of stretch in pixels in the finished picture, typically 1
W_P	width of the finished picture in pixels, typically 600 to 3600
F	focal length of the camera lens, in mm
W_S	width of the sensor, also in mm
C	cropping, fraction of the picture actually used
δ	declination of the objecting being photographed

Table 1: Variables used in the formulae

typical. A whole computer screen (not an exceptionally large one) is typically 1600 pixels wide; a poster-size print (36 inches at 100 dots per inch) could be 3600.

The other factors that matter are the focal length of the lens (F) and the width of the sensor (W_S). These are both in millimeters.

We can also allow for cropping in case you're going to enlarge just part of the picture. Call that C, the fraction of the picture you're using. If, for example, your finished picture is just half the width of what the sensor captured, $C = 0.5$.

And it also matters what part of the sky you're photographing because the effect of the earth's rotation is smaller near the poles. Let δ be the declination of the objecting being photographed.

Table 1 sums these up.

DSLR "crop factor"

Note that the "crop factor" of a DSLR (typically 1.5 or 1.6 for APS-C sensors) does not figure into any of this. Our formulae use the true size of the sensor (24 mm wide for a typical APS-C sensor, 36 mm for a full-frame sensor).

When APS-C DSLRs were introduced, the "crop factor" was a way to tell photographers they were getting a smaller field of view than with film. Instead of saying the field of view was smaller, people said the picture was bigger, that the lens focal length was effectively longer.

Actually, "crop factor" is just relative sensor size. It is not an optical effect and does not change the effective focal length of the lens. (A teleconverter does.) There is nothing optically fundamental about the "full-frame" sensor (the size of 35-mm film); a sensor can be any size, and focal length is not defined in terms of the sensor.

I should add that the focal length marked on lenses for APS-C DSLRs is the true focal length; it is not adjusted for sensor size.

Arc-seconds per pixel on the finished picture

The key quantity we need to know is the angular size, in the sky, that corresponds to one pixel on the finished picture. We will then need to figure out how long it takes the sky to move that far as the earth rotates.

The width of the finished picture *on the sensor,* in millimeters, is $W_S \times C$. That same width, in pixels, is W_P. So one pixel in the finished picture corresponds to $W_S C / W_P$ millimeters on the sensor. Call that distance x.

A distance of x millimeters on the sensor corresponds to an angle of x/F radians in the sky (using the small-angle approximation to the arctangent function). That is $206\,265\ x/F$ arc-seconds.

The sky rotates 15 arc-seconds per second of time at the celestial equator. So, at the celestial equator, one pixel on the finished picture, corresponding to x millimeters on the sensor, is the amount of smearing you would get in

$$\frac{206\,265\ x}{15\ F}\ \text{ seconds}$$

but we're not finished. We might want to tolerate more than one pixel of smearing, so let's put in S. And we haven't taken into account the declination, which slows down the angular movement by a factor of $\cos\delta$. That gives us

$$\frac{206\,265\ x\ S}{15\ F\cos\delta}\ \text{ seconds}$$

and we have yet to plug in the definition of x. Doing that, and also dividing

206 265 by 15, we get

$$\frac{206\,265\ (W_S C/W_P)\ S}{15\ F \cos\delta} = \frac{13\,751\ W_S\ C\ S}{W_P\ F \cos\delta} \text{ seconds.}$$

Simplifying

That formula is complicated! Can we simplify it for a typical case? Yes. Typically you know and care about W_S (sensor size), W_P (width of finished picture in pixels), and F (focal length). The acceptable smear, S, can be assumed to be 1 pixel. You can also assume that $\cos\delta$ is close to 1 (here we assume 0.9). Making those assumptions, we get a simpler formula:

$$\frac{13\,751 \times W_S \times 1 \times 1}{W_P \times F \times 0.9} = \frac{15\,279\ W_S\ C}{W_P\ F} \text{ seconds.}$$

That's not bad, but still not the "Rule of 500." Let's make even more assumptions. Suppose you're using an APS-C sensor and not cropping the picture, and that you're going to display it on the full screen of your computer. Then:

$$\text{Maximum exposure} = \frac{15\,279 \times 24 \times 1}{1600 \times F} = \frac{230}{F} \text{ seconds.}$$

That's a "Rule of 230." Why not 500? Two reasons. We're viewing the image rather large (full-screen-size) and we're using an APS-C sensor. With a full-frame sensor, that 230 would be 345, and with a tiny bit more tolerance of smear in the picture, you could easily get to 500.

The most important point to remember

The most important point to remember is that this rule is just an approximation. It's not as though you'll get perfect pictures at $499/F$ and terrible blurs at $501/F$. it's just a starting point.

Appendix: Guiding tolerance

What if you're doing guided astrophotography and need to know how well you must guide? Recall that we decided the acceptable smear, in arc-seconds, was $206\,265\ x/F$, where $x = W_S C/W_P$. Plugging that in:

$$\text{Guiding tolerance} = \frac{206\,265\ W_S\ C}{W_P\ F}\ \ \text{arc-seconds.}$$

Consider, again, a concrete case. With an APS-C DSLR sensor, and displaying the uncropped picture to fill a typical comptuer screen, we have

$$\text{Guiding tolerance} = \frac{206\,265 \times 24 \times 1}{1600 \times F} = \frac{3000}{F}\ \ \text{arc-seconds}$$

where, for ease of memory, 3000 was rounded slightly from 3094. That means that with a 300-mm lens, your tolerance is 10 arc-seconds. Now is that peak-to-peak or RMS? Hard to say, but probably RMS.

This also explains why we seldom do deep-sky work with focal lengths longer than 2000 mm, where our typical long-term autoguiding accuracy (1.5 arc-seconds RMS) equals the guiding tolerance in the finished picture. However, there's a further trick to it: With pictures taken at longer focal lengths, the star images are likely to be appreciably more than 1 pixel in diameter even in the finished picture, and more smear may be tolerable.